SAR Image Processing Using Super Resolution Spectral Estimation with Annihilating Filter

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Abstract – **In this work, the problem of scatterer separation in SAR imaging is considered. We discuss a solution based on the annihilating filter method (AFM), which allows joint estimation of range and complex reflectivity of targets within a resolution cell. We show that with Cadzow denoising procedure, the algorithm produces good frequency and amplitude estimation results. The performance of the algorithm compares favorably with the performance of DFT and MUSIC.**

Keywords-component; SAR Imaging, Super Resolution Technique, Annihilating Filter Method (AFM), Cadzow Denoising Procedure.

I. INTRODUCTION

Synthetic aperture radar (SAR) performs a large number of functions which range from detection and discrimination of targets to mapping large areas of ground terrain. SAR moves along its path, it transmits pulses at microwave frequencies at uniform pulse repetition interval and measures the echo signals. The radar can produce high resolution two-dimensional and even three-dimensional imagery of the ground surface. The quality of ground maps generated by SAR is determined by the size of the resolution cell, which is specified by range and azimuth resolutions of the system [1].

SAR images are obtained basically using the Fourier transform and interpolation. For target separation within a resolution cell, however, high resolution methods should be applied based on modern spectral estimation techniques. In [2] the MUSIC algorithm was proposed to enhance target separability in range and azimuth inside a resolution cell. Two main drawbacks appear in application of this method in SAR imaging. As the MUSIC algorithm was initially proposed to estimate the direction of arrival of signals, its response does not provide true information about the target backscattering power. For this reason, a SAR image with high quality cannot be achieved by using of MUSIC alone. The second drawback is that, the MUSIC algorithm exploits the data covariance matrix in order to build up the noise subspace and estimate signal sources locations. Multiple acquisitions are required to obtain the signal statistics which is not appropriate for a single SAR survey. Thus, it has been shown that the MUSIC technique is not well suited for a complete 2D super-resolution SAR image. The aim of this work is to investigate super-resolution spectral estimation with the annihilating filter applied to SAR imaging to improve the target resolution within a resolution cell. In presence of noise, a larger amount of data has to be considered and the singular value decomposition with Cadzow denoising algorithm is used.

The rest of this paper is organized as follows. In section II, annihilating filter method is introduced and Cadzow denoising procedure is described for the case of low SNR. Use of the annihilating filter method for the SAR imaging is described in Section III. Performances are demonstrated by the simulation results in Section IV. Finally, Section V draws our conclusions.

II. ANNIHILATING FILTER METHOD

The annihilating filter method is mostly developed in the area of parametric spectral estimation where the problem is to estimate the parameters of a linear combination of complex exponentials from a set of measurements. This idea is also employed by Vetterli et al. to devise a sampling method for signals with finite rate of innovation [3].

Consider the sampled signal S[n] which consists of K exponentials

$$S[n] = \sum_{i=0}^{K-1} c_i u_i^{n} \quad (1)$$

in which an amplitude $c_i \in \mathbb{R}$, a complex sinusoid $u_i = e^{j2\pi f_i}$, and n is a sampled index. A filter A[n] is called the annihilating filter of the signal S[n] when $(A * S)[n] = 0 \ \forall\, n \in \mathbb{N}$. The signal S[n] is annihilated by the filter

$$A(z) = \prod_{i=0}^{K-1} (1 - u_i z^{-1}) = \sum_{l=0}^{K} A[l] z^{-l} \quad (2)$$

The annihilating filter method consists of finding the values of c_i and u_i in S[n] and is composed of three parts: first, we need to find the annihilating filter that involves solving a linear system of equations. In the matrix form,

$$\begin{bmatrix} \vdots & \vdots & \cdots & \vdots \\ S[K] & S[K-1] & \cdots & S[0] \\ S[K+1] & S[K] & \cdots & S[1] \\ \vdots & \vdots & \ddots & \vdots \\ S[2K+1] & S[2K-2] & \cdots & S[K-1] \\ \vdots & \vdots & \cdots & \vdots \end{bmatrix} \begin{pmatrix} A[0] \\ A[1] \\ \vdots \\ A[K] \end{pmatrix} = 0 \quad (3)$$

The number of measurement S[n] has to be larger than 2K. If the number of measurement S[n] is 2K, rank of S has

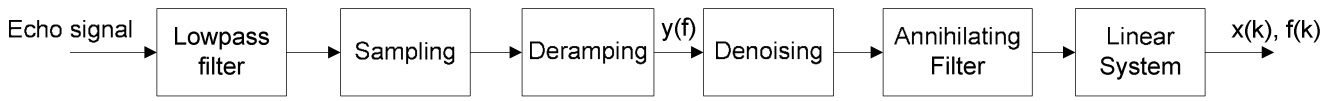

Fig.1 SAR Doppler processing scheme with the annihilating filter.

to be K.

Second, we find u_i. Once the filter coefficients A[n] are found from (3), the values u_i are the roots of the annihilating filter A(z) (2) where $u_i = e^{j2\pi f_i}$.

Third, we estimate the weights c_i. We can find c_i from the matrix U which is a Vandermonde matrix :

$$U = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ u_0 & u_1 & \cdots & u_{K-1} \\ \vdots & \vdots & \cdots & \vdots \\ u_0^{K-1} & u_1^{K-1} & \ddots & u_{K-1}^{K-1} \end{bmatrix} \quad \text{and}$$

$$\begin{bmatrix} 1 & 1 & \cdots & 1 \\ u_0 & u_1 & \cdots & u_{K-1} \\ \vdots & \vdots & \cdots & \vdots \\ u_0^{K-1} & u_1^{K-1} & \ddots & u_{K-1}^{K-1} \end{bmatrix} \begin{pmatrix} c_0 \\ c_1 \\ \vdots \\ c_{K-1} \end{pmatrix} = \begin{pmatrix} S[0] \\ S[1] \\ \vdots \\ S[K-1] \end{pmatrix} \qquad (4)$$

This Vandermonde system has a unique solution when $u_p \neq u_q, \ \forall p \neq q$.

If noise is not included in discrete-time measurements of the signal, K exponential terms can be found using the annihilating filter method. At low SNR, the singular value decomposition is used. However, this method is ill-conditioned because root-finding is itself not at all robust to noise. This problem can be solved using the Cadzow denoising algorithm. This iterative procedure was suggested by Tufts and Kumaresan in [4] and analyzed in [5].

The Cadzow iterative denoising algorithm process is as follows. First of all, build Toeplitz matrix T of dimension L X (L+1) using S[n] as (3). Second, set ε to be a small constant. Third, while $(\sigma_{K+1})/(\sigma_K) \geq \varepsilon$, where σ_K is the K-th singular value, enforce rank K on T by setting the L-K+1 smallest singular values to zero, and enforce the Toeplitz form on T by averaging the coefficients along the diagonals. Finally extract the denoised spectral coefficients from the first row and first column of T.

The computational cost of the Cadzow denoising algorithm is higher than the annihilating filter method since it requires performing the SVD of a square matrix of large size, typically half the number of samples. If ε is smaller, more computation is needed. However, modern processors can perform the SVD of a square matrix fast enough.

III. Annihilating Filter Method Applied For SAR Imaging

In radar, range resolution is accomplished through range gating and pulse compression techniques. The azimuth resolution is determined by antenna size and radar wavelength and enhanced by the presence of the synthetic aperture. We show how one can modify standard SAR Doppler processing to improve azimuth resolution using the annihilating filter method.

We consider baseband components of combined returns for the resolution cell as the sum of K individual returns reflected by each target within the cell. Let the components are filtered with a lowpass filter, sampled uniformly above the Nyquist rate and deramped. Then, the output signal is the linear combination of K complex exponentials. This signal is completely determined by the knowledge of the K amplitudes and the K frequencies. In respect that the echo signals are in noise we apply the iterative denoising algorithm and the annihilating filter to estimate the amplitudes and frequencies of the deramped signal. Thus, we solve the problem of positions and weights estimation of K Diracs of period F in frequency domain (Fig.1):

$$y(f) = \sum_{k=1}^{K} \sum_{n \in Z} x_k \delta(f - f_k - nF) + \epsilon(f) \qquad (5)$$

IV. Simulation Results

We have simulated the annihilating filter method with the Cadzow denoising algorithm applied to the SAR Doppler processing and investigated their resolution capabilities with a comparative analysis of the super-resolution MUSIC algorithm and DFT.

A. One Signal Case

A sampled signal consisting of a single complex exponential in zero mean white Gaussian noise is

$$S(n) = c \times e^{j2\pi f_1 n} + e(n) \qquad (6)$$

For comparison purposes, 500 runs of Monte Carlo simulation at sampling rate of 10kHz has been conducted. The frequency of the signal was chosen to be $f_1 = 1 + x$ (kHz), where x is uniformly distributed in the interval [0,0.1].

Fig. 2 and 3 show the root mean square error (RMSE) of estimated frequency and amplitude, respectively, obtained using the AFM with and without Cadzow denoising along with MUSIC (for frequency estimation) and DFT. The SNR varies from -10dB to 30dB. The search range of MUSIC comprises [0:0.1:3000] Hz. The search range is driven by the range of Doppler frequency in SAR imaging. The results indicate that the AFM with Cadzow denoising is fairly reliable and compares favorably with MUSIC and DFT.

It is well known that oversampling the input signal in DFT does not improve ability to resolve two closely spaced signals in the frequency domain. However, in AFM increasing the signal sampling rate gives better resolution performance. It can be explained by more accurate finding the roots of the polynomial (2). In MUSIC, the resolution ability doesn't increase as sampling rate goes high because of limited search bin size.

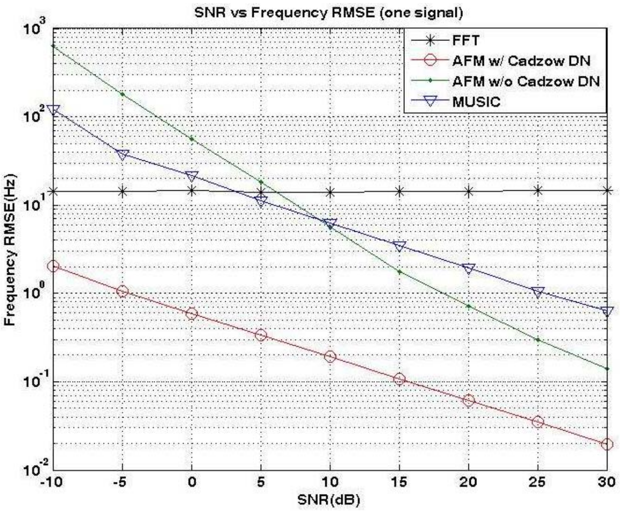

Fig.2 SNR vs Estimated Frequency RMSE in one signal case

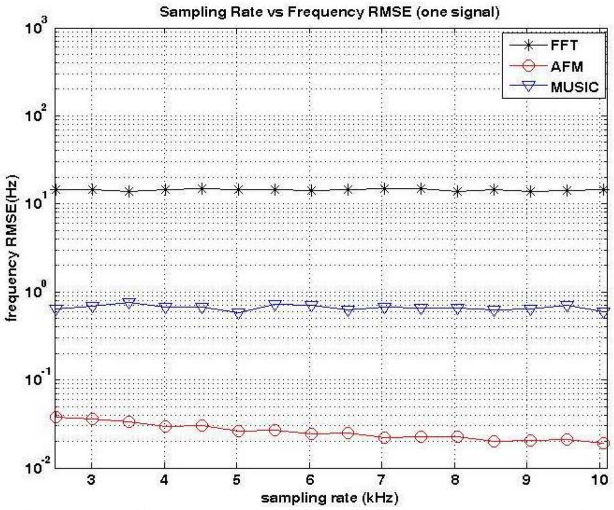

Fig.4 Sampling Rate vs Estimated Frequency RMSE in one signal case

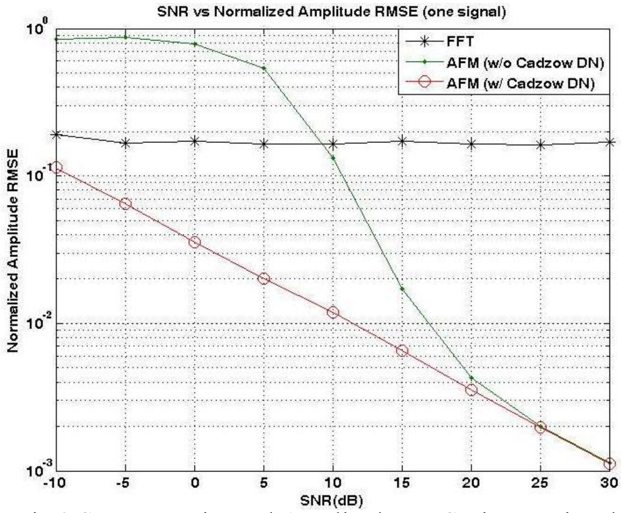

Fig.3 SNR vs Estimated Amplitude RMSE in one signal case

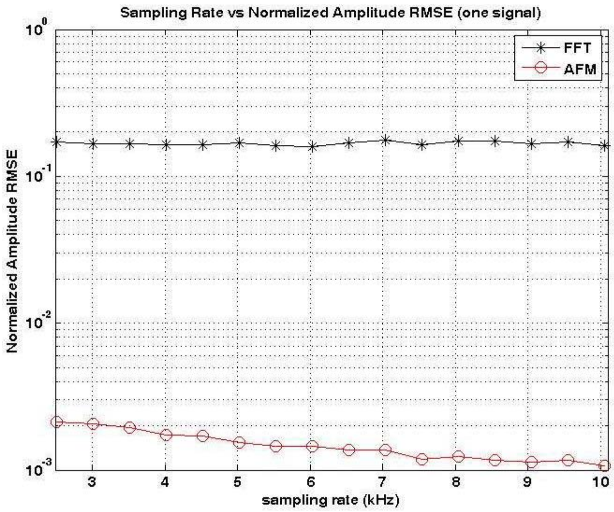

Fig.5 Sampling Rate vs Estimated Amplitude RMSE in one signal case

Fig. 4, 5, 8, and 9 show the root mean square error (RMSE) of estimated frequency and amplitude as sampling rate varies from 2.5kHz to 10kHz. The errors obtained using AFM decrease as the sampling rate is higher.

B. Two Signal Case

In two signals scenario we have the sampled data which is a superposition of two exponentials distorted by zero mean white Gaussian noise:

$$S(n) = c_1 \times e^{j2\pi f_1 n} + c_2 \times e^{j2\pi f_2 n} + e(n) \qquad (7)$$

The frequencies of the signals were chosen to be $f_1 = 0.8 + x$ (kHz), x is uniformly distributed in the interval [0,0.4] and $f_2 = f_1 + 0.021$ (kHz). The separation 21 Hz is less than $1/T_{ob}$, the resolution limit of Fourier based spectral estimation methods, where T_{ob} = 40 ms. The other simulation parameters are same as in one signal scenario. The results of 500 sample simulations obtained for the AFM, MUSIC and DFT are shown in Fig. 6, Fig. 7, Fig. 8 and Fig. 9.

Fig. 6 and 7 show the root mean square error (RMSE) of estimated frequency and amplitude as SNR varies from -10dB to 70dB. As can be seen, AFM shows a better performance as SNR goes higher. In low SNR (less than 0 dB), however, AFM has a high frequency RMSE. This is due to the fact that iterative Cadzow denoising reduces not only noise components, but also the signal components at the high levels of noise. As the amplitudes are calculated with use of the frequency estimates, the amplitude estimates have higher RMSE at low SNR as well.

V. CONCLUSION

In this work, it has been demonstrated that it is possible to improve target separation within a resolution cell for SAR imaging with use of the Annihilating Filter Method (AFM)

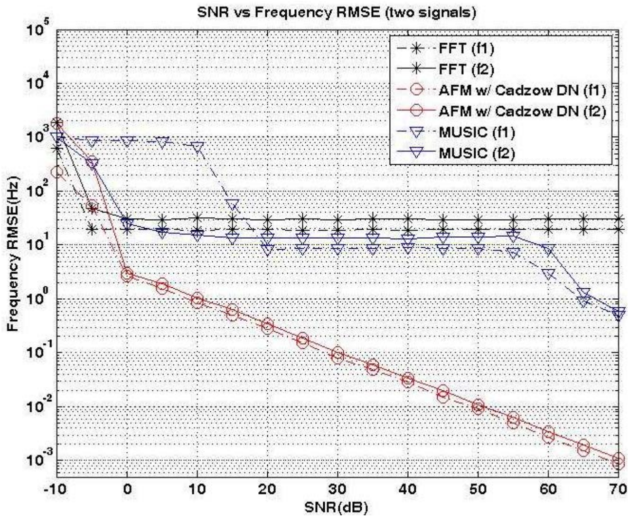

Fig.6 SNR vs Estimated Frequency RMSE in two signals case

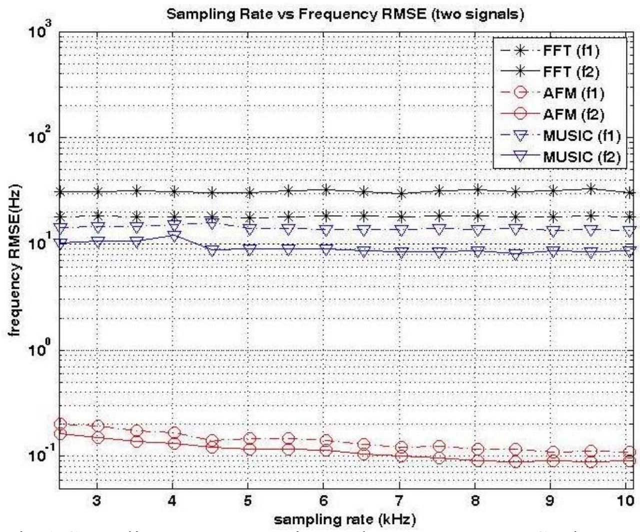

Fig.8 Sampling Rate vs Estimated Frequency RMSE in two signals case

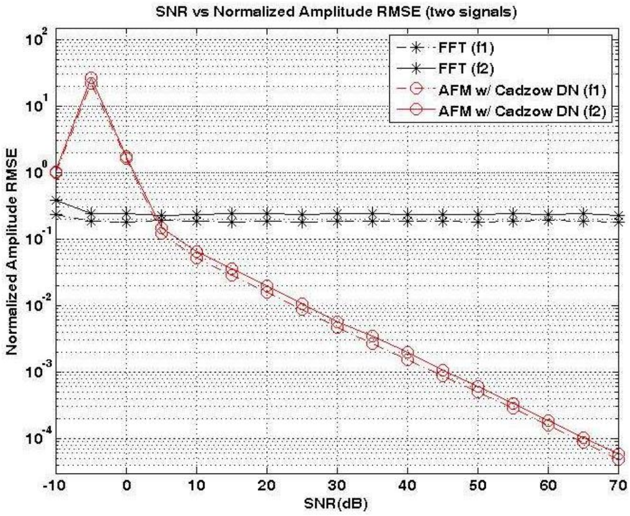

Fig.7 SNR vs Estimated Amplitude RMSE in two signals case

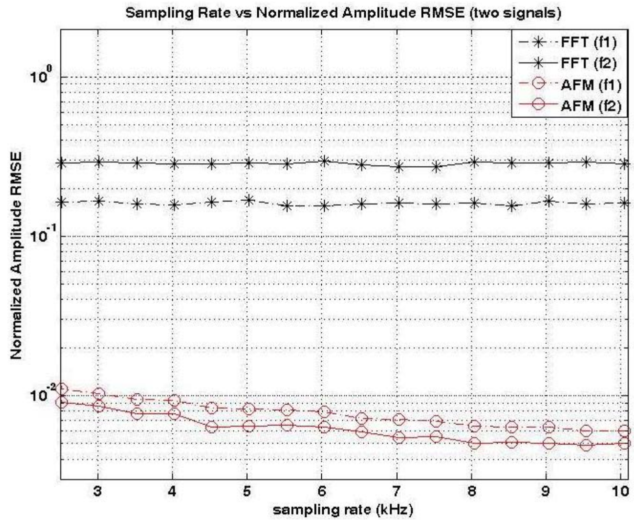

Fig.9 Sampling Rate vs Estimated Amplitude RMSE in two signals case

and the Cadzow denoising algorithm. Specificity of SAR Doppler processing of noisy echo signals allows utilizing AFM to enhance SAR imaging resolution. AFM shows better performance than super-resolution MUSIC algorithm. Besides, MUSIC cannot provide with true information about signal amplitudes. In addition, multiple acquisitions are required in MUSIC to obtain the signal statistics which is not appropriate for a single SAR survey.

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